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Derivation of Demand Elasticities from Travel Choice Elasticities

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Abstract

We derive ordinary Marshallian demand elasticities from mode choice elasticities. The fact that purely choice elasticities are conditional on a fixed number of trips is used to build the trip generation part of total travel demand. The generation elasticity for all modes in the group is the difference between ordinary and choice elasticities. Demand elasticities incorporate 'pure substitution' and 'money expenditure' effects, relating conditional demand to ordinary demand. Choice elasticities replace conditional demand elasticities and a correction compensates for the choice estimation method. The result obviates the error of treating choice elasticities as market demand elasticities.

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1.0 Introduction

The problem with a price-choice elasticity is that it cannot be used validly or reliably to estimate market response to a price change. It may provide a reasonable approximation for a minor choice, but for anything with a large market share the choice elasticity will underestimate the demand response. The household motor fuel study by Smith *et al.* (2010) found the choice elasticities to be reasonable approximations to the ordinary own-price elasticities of demand for the fuels with market shares of less than 10 per cent, but the own-elasticity of demand for petrol (90 per cent share of the market) was underestimated by approximately 80 per cent.

Demand analysis is conventionally formulated in terms of the average consumer who buys varying quantities of each item. However, Brown and Deaton (1972) noted that a per capita model '...could not distinguish a good which was extremely income elastic but which was consumed evenly throughout the population from a good, perhaps newly introduced, the consumption of which was rapidly spreading among the consuming public'.

Household consumption surveys are at a detailed level, but generally each household has consumed some of each class of goods. At a more detailed level, the choices are between items. However, analysis at some level of aggregation is generally convenient, and disaggregate choice modelling has mainly been reserved for large items or travel choices which are mutually exclusive. There is no clear boundary. Transport specialists familiar with discrete choice analysis recognise the importance of 'generation', the combined demand for the group of commodities between which the choices have been analysed, but have found it difficult to devise satisfactory ways of measuring it (Oum *et al.*, 1992).

The total response to a change in price may be decomposed into three effects (Novshek and Sonnenschein, 1979):

1. The aggregate substitution effect — with real income held constant an increase in the price of transport will cause consumers to substitute away from transport. This will only affect the individuals who currently choose the mode of transport for which the price is increased.
2. The aggregate income effect — an increase in price of a mode of transport will reduce the real income for travellers by that mode. Normally this means that the consumer will need to adjust expenditure shares on all consumption.
3. The change in commodity effect or mode switching — an increase in the price of one mode will cause some users to change to an alternative mode. Provided the change in price is not extreme, only a small proportion of the population will change modes.

The discrete choice model provides elasticity estimates for the third effect, and the central issue in this paper arises from the fact that a discrete choice model usually deals with a narrowly defined set of alternatives. These are distinguished mainly by specific attributes rather than being generically different, as in continuous demand for broad commodity categories. The choice decision is concentrated on the substitution between alternatives (effect 3), rather than on demand in the broader sense where both the substitution between broad categories of commodities (effect 1) and the budgetary or income impacts (effect 2) of these demand decisions are taken into account.

The only recognised response to a price change in a purely choice model is substitution of one mode for another, implying that total travel remains constant. In conditional demand terms, the condition is that the quantity of travel is fixed. This differs from Pollak's (1969) concept in that the *conditional demand* here is subject to a quantity constraint, not a constraint on expenditure.

One way that has been used to overcome the deficiency that discrete choice models just give 'choice-elasticities', rather than demand elasticities, is to link a choice model to a continuous demand model in such a manner that each can specialise on its own strength while relying on the other to supplement for its lack of coverage. If preferences have been revealed in actual consumption, the modelling problem can be approached with a discrete-continuous model, where there is both a discrete choice and a decision to consume a certain quantity of the chosen item or of the service it provides. Dubin and McFadden (1984) modelled the discrete choice of an electrical appliance in conjunction with a continuous choice of how much to use the services of the chosen item. Berry *et al*. (1995 and 2004) modelled household choice and then aggregated to obtain product level demands, thus analysing both micro data on household choices and aggregate data on product-level demands in one consistent framework. Hanemann (1984) dealt with the case of substitutes, such as alternative brands, where one brand is chosen but the quantity varies. The method has been refined by Bhat (2008); another discrete-continuous application is by Fang (2008). However, the large number of studies that are purely choice — using some form of additive in income random utility model (AIRUM) — indicates that a method of deriving demand from choice elasticities is still needed. This task is the topic of the paper.

Inference in the other direction is relatively straightforward; it was shown by Taplin (1982) that ordinary demand elasticities can be decomposed into choice and generation components, but the transformation cannot be reversed. The focus on travel in the derivation arose from recognising that aggregate travel demands are based on individual mode choices plus a travel generation effect, so that there were no qualms about inferring from nominally continuous relationships to choice or share elasticities. The relationship was generalised by recognising that choice between unlike items can be represented in terms of expenditure choice elasticities, also derived from ordinary demand elasticities (Taplin and Smith, 1998).

In practice, data of differing types lead to contrasting estimation methods. Discrete choice estimation uses individual data, whereas household demand estimation generally uses data which has been aggregated to some extent (Stone, 1954; Clements and Theil, 1996). Our objective is to close the gap between choice and demand elasticities for travel with an approximation that is sufficiently robust to forecast market demand responses.

2.0 Demand and Choice

For many data sets containing revealed or stated choices, no associated or extraneous information on travel expenditure or numbers of trips is available, thus inhibiting the use of comprehensive econometric models. However, the nature of the data does not prevent the analyst from modelling travel demand in terms of two-stage budgeting and investigating the relationship between mode choice and conditional demand. Conditional demands are formulated when the individual's consumption decision for all other goods is notionally held constant and the analyst studies a small section of consumption (travel) without including the relative prices of the unrelated goods (Pollak, 1969).

From a different perspective, Quandt (1968) compared elasticities estimated by time series and those estimated by discrete choice, the mode choice elasticities being ordinary elasticities with a term to hold demand for a travel constant. This also is conditional demand, but the level of transport consumption is fixed. It differs from the traditional conditional demand approach, where the fixed or 'pre-allocated' good is non-transport, so that the consumer determines the amount of non-transport to consume at current prices and income (Pollak, 1969, 1971). The conditional demands for transport alternatives are functions of the allocation of money to the transport sector and the prices of the transport alternatives. The formulation of conditional demands to investigate substitution between closely competing goods is widely used in agricultural economics (Boonsaeng and Wohlgenant, 2009; Muhammad and Hanson, 2009) and has also been used for estimating fuel demand elasticities (Gundimeda and Köhlin, 2008). The relationship between conditional and unconditional demands has been derived in the continuous conditional demand literature (Edgerton, 1997; Carpentier and Guyomard, 2001).

As noted in Section 1, the discrete choice model provides elasticity estimates for mode switching but not for aggregate substitution to or from transport, nor for the effect of a transport price change on real income. The consumer is assumed to have optimally chosen the level of expenditure allocated to transport in order to allow the desired consumption of non-transport goods. Choice studies do not go on to estimate demand functions for the non-transport sector of expenditure. However, there is evidence that transport expenditure affects spending behaviour on non-transport goods (Ferdous *et al*., 2010; Ma *et al*., 2011). The demand system approach is often called conditional logit (after the elemental choice model, the multinomial logit) or, more generally, the conditional choice model. The conditional choice model is used to estimate choice elasticities from revealed preference data (Koppelman *et al*., 2001) or combined stated and revealed preference data (Hensher, 2008).

2.1 The market demand setting

In a large population of consumers choosing between a number of travel modes, there is potential to consume a mixture of modes, but most choose just one. The level of demand for each transport alternative is divisible, being accommodated by the choice of where to travel and how frequently. The unit measure of travel may well be *personal travel kilometres*. An increase in the price of the currently chosen mode can have two effects. First, the consumer may choose to continue travelling by the current mode, in which case their demand is governed by the substitution between travel and all other goods as well as any income effect. Alternatively, the consumer may switch to another mode. As before, the consumer will reconsider the allocation of expenditure between all other goods and travel.

The travel demand elasticities will differ from the mode-share elasticities in two important ways. First, the own-price demand elasticity is smaller[1] (more negative) than the corresponding choice elasticity, because it includes a price effect for those who do

[1]It is assumed that travel is not a Giffen good.

not switch. Second, the cross-price demand elasticities are generally smaller (less positive) than the corresponding choice elasticities, because there is the potential for a traveller who does switch also to adjust their level of travel demand. The cross-price elasticity cannot be less than zero, because the individuals who are travelling by modes that have not had a price increase will continue to maintain their level of demand.

If the price of the current travel mode increases, then the consumer may respond by absorbing the price increase and forgoing some non-travel consumption: a fixed demand for travel. This represents the situation where the conditional demand function (the condition being a fixed quantity of travel) is equal to the unconditional demand function. Alternatively, the consumer may reallocate their budget between travel and non-travel at the new price, adjusting trip frequency, travel type, or destinations, as well as other consumption. This paper presents a way in which this behaviour may be captured in conditional and unconditional elasticities. As already noted, the theory differs from the traditional approach because the *conditional demand* here is subject to a quantity constraint instead of being subject to an expenditure constraint (Pollak, 1969). The purpose of treating choice modelling as conditional analysis is to deal with limited data; where there are no or few data limitations, other methods can be applied, as will be briefly reviewed in Section 2.2.

2.2 Discrete/continuous demand and market equilibrium models

In a discrete/continuous choice model of the type used by Hanemann (1984), the consumer is assumed simultaneously to select one good from a group of substitutes and the quantity of that good to consume. Other examples include brand choice (Chiang, 1991; Chintagunta, 1993), appliance choice, and energy demand (Dubin and McFadden, 1984). The choice component is an initial investment in a technology. Another class of model for investigating consumer choices over time is the discrete/continuous demand model and subscription to and use of a service (Madden *et al*., 1993). In transport research, the setting has been applied to vehicle ownership and usage (Train, 1986; Hensher *et al*., 1992), where the continuous part of the estimation is supported by vehicle usage data. Each of these studies models the ownership and changeover of vehicles within a household as well as the kilometres. Whereas Hensher *et al*. maintained a four-year panel, Train's study used a cross-sectional sample with a question about usage and ownership of vehicles for the previous 12 months.

Berry *et al*. (1995 and 2004) model household choice of motor vehicle where there are many choices, then aggregate to obtain product level demands. Estimation of some parameters is based on the micro data, but estimation of elasticities for differentiated products requires assumptions of the sort used in market-level data. Unobserved random coefficients are needed to describe the relatively tight substitution patterns that are found in the data, and second-choice data help to obtain estimates of the parameters that govern these substitution patterns. Berry *et al*. (2004) comment that outside information or cross-sectional variation in choice sets must be used to pin down the absolute level of elasticities.

Bhat and Sen (2006) relax the perfect substitution assumption, noting that households with more than one vehicle may use different vehicles for different purposes. Their model analyses the holdings and use of multiple vehicle types by households, supported by annual vehicle mile estimates. Rajagopalan and Srinivasan (2008) study the correlates between social demographic variables, the joint mode choice, and the mode expenditure decision.

Kockelman and Krishnamurthy (2004) incorporate the mode choice (limited to private vehicle and other) into the discrete-continuous setting. They approach the problem by using a two-stage estimation procedure where the *prices* of the alternatives are obtained in the discrete choice setting and passed on to the continuous model; their model uses a two-day trip diary.

All of these studies deal with the same underlying problem as the one considered in this paper, but are not limited to the results of purely choice studies.

3.0 Decomposition of Demand into Item Choice and Generation

A set of unconditional demands for a group of competing transport modes embodies the choices between them as well as the travel group demand generation elements. Although the title of the Taplin (1982) paper was 'Inferring ordinary elasticities from choice or mode-split elasticities', it first derived choice from demand elasticities and then showed that extraneous information is needed to reverse the inference. The choice elasticities are extracted by deriving matrix M of choice or share elasticities m_{ij} from the corresponding matrix E of travel demand elasticities e_{ij} thus:

$$M = (I - S)E. \tag{1}$$

The identity matrix is I and S comprises the row vector of n physical shares s_i repeated n times. Thus the element m_{ij} of M is:

$$m_{ij} = -s_1 e_{1j} - s_2 e_{2j} - \cdots + (1 - s_i)e_{ij} - \cdots - s_n e_{nj}.$$

3.1 The demand generation component

From equation (1), the element

$$(E - M)_{ij} = (SE)_{ij} = \sum_{k=1}^{J} S_k E_{kj} \text{ (the rows of } S \text{ being identical)}$$

$$= \eta_j \text{ for each } i. \tag{2}$$

This is the travel demand generation elasticity with respect to price j.

A more general expression of demand elasticity e_{ij} in terms of choice elasticity m_{ij} takes account of the effect of the change in an individual transport mode cost or price p_{jT} on the total transport demand q^T thus:

$$e_{ij} = m_{ij} + \frac{\partial \log q^T}{\partial \log p_{jT}}. \tag{3}$$

If it is assumed that η_j is equal to the second term on the right-hand side of equation (3), then this would imply $\partial \log s_i / \partial \log q^T = 0$. However, this is true for the homothetic case only, and an additional correction term is required to make the relationship general.

The problem is to estimate demand elasticity e_{ij} when choice elasticity m_{ij} has already been estimated by one of the well-known choice modelling methods. The path to the estimation of the required terms is via the expenditure generation elasticity and conditional demands.

4.0 Decomposition into Expenditure Choice and Generation

The demand elasticities can also be decomposed in terms of expenditure. Matrix F of expenditure elasticities is simply obtained from the matrix of ordinary elasticities: $F = E + I$. Taplin and Smith (1998) derived matrix G of expenditure choice elasticities from F and therefore from the ordinary demand elasticities E thus:

$$G = (I - W')F, \tag{4}$$

where W' is formed by n repetitions of the row vector of expenditure shares w'_j within the travel group T. The cross-price expenditure elasticities in F are identical to the corresponding ordinary elasticities, but each own-price expenditure elasticity takes a price change into account at two levels. The ordinary quantity demand response is substantially offset by the price change increasing or decreasing the expenditure on each item purchased. Thus an own-price expenditure elasticity differs by 1.0 from the corresponding ordinary elasticity.

The elements of the expenditure choice matrix G account only for the substitution effects within the group, leaving the effects of the individual prices on group expenditure to be represented by the travel group generation elasticities. The matrix of expenditure generation elasticities (Taplin and Smith, 1998) is:

$$\Omega = W'(E + I). \tag{5}$$

An individual elasticity ω_j in Ω with respect to price j is:

$$\omega_j = \sum_k w'_k e_{kj} + w'_j. \tag{6}$$

This is the expenditure generation elasticity for the whole group with respect to the price of j.

4.1 Generation elasticity and conditional demand

The expenditure generation elasticity for the travel group T can be incorporated into the conditional demand system which describes demands for the group items given a pre-established expenditure group. The conditional income elasticity e'_i represents the amount of an additional budget allocation to group T that would be apportioned to good i: $e'_i = e_i/E_T$, where E_T is the group income elasticity. The conditional expenditure share $w'_i = w_i/W_T$, where W_T is the group share of total expenditure.

Ordinary elasticity of demand e_{ij} may be decomposed into a conditional demand response and an allocation of money within the group (Pollak, 1969):

$$e_{ij} = e'_{ij} + \omega_j e'_i. \tag{7}$$

The first element on the right-hand side of equation (7) is the conditional elasticity of demand e'_{ij} for transport mode i with respect to the price of mode j — where i and j belong to group T (Pollak's 'pure substitution effect'). The total expenditure for group T is fixed, because the budget is fixed and the quantities of all items not belonging to T are pre-allocated. The second element is a group expenditure effect $\omega_j e'_i$ (Pollak's 'money expenditure effect') where generation elasticity ω_j is the elasticity of expenditure for group T with respect to the price of mode j.

Substituting equation (6) into equation (7) gives:

$$e_{ij} = e'_{ij} + \left(\sum_k w' e_{kj} + w'_j\right) e'_i. \tag{8}$$

It is shown in Appendix 1 that if the transport group is weakly separable, then equation (8) can be expanded into:

$$e_{ij} = e'_{ij} + w'_j (E_{TT} e'_j + 1) e'_i + w'_j W_T E_T e'_i (e'_j - 1), \tag{9}$$

where E_{TT} is price elasticity of demand for the group, E_T is the group income elasticity, and W_T is group share of total expenditure.[2]

5.0 Ordinary Demand Elasticity

If an estimate of the price elasticity of demand for the group E_{TT} is available, then it is used in the derivation of the ordinary demand elasticities e_{ij}. Otherwise, E_{TT} is inferred by way of Frisch's (1959) money flexibility ϕ, which is the inverse of the elasticity of the marginal utility of income. In either case, an estimate of the group income elasticity E_T is also required.

5.1 Inferring group demand elasticity

It has been shown by Deaton and Muellbauer (1980) that for additive preferences, the own-price demand elasticity for group i is:

$$E_{ii} = \phi E_i - E_i W_i (1 + \phi E_i).$$

For transport this becomes $E_{TT} = \phi E_T - W_T E_T (1 + \phi E_T)$, which can be expressed as:

$$E_{TT} = \phi E_T - \phi W_T E_T E_T - W_T E_T, \tag{10}$$

where W_T is the proportion of income allocated to transport. Noting that $w_j = w'_j W_T$, substitution of equation (10) into equation (9) gives:

$$e_{ij} = e'_{ij} + w'_j [(\phi E_T - \phi W_T E_T E_T - W_T E_T) e'_j + 1] e'_i + w_j E_T e'_i (e'_j - 1). \tag{11}$$

5.2 Incorporating the choice elasticity

In Appendix 2, it is shown that the following relationship holds generally. The primes have been added to the choice elasticities to indicate that they are conditional in Pollak's (1969) sense:

$$e'_{ij} = m'_{ij} - \sum_k w' m'_{kj} - w'_j. \tag{12}$$

[2]This result has also been obtained by Carpentier and Guyomard (2001).

The expression $\sum_k w'_k m'_{kj} - w'_j$ has a similar form to the right-hand side of equation (6) and may be thought of as a conditional expenditure generation elasticity ω'_j representing the additional budget allocated to the group in order to keep the total quantity fixed, should the price of mode j increase. This generation elasticity comprises two parts: w_j adjusts for the change in expenditure should the traveller continue to consume the same amount of travel by mode j before and after the price change. The other part accounts for the possibility of switching between modes. The whole adjustment $\sum_k w'_k m'_{kj} - w'_j$ is a correction for the method of representing switching behaviour in the estimation of choice elasticities.

The method of inferring an individual demand elasticity e_{ij} from choice elasticities depends upon the source of the group demand elasticity E_{TT}.

5.2.1 Group price elasticity known

If an estimate of E_{TT} is available, then equation (12) is substituted into equation (9) to give the estimating equation:

$$e_{ij} = m'_{ij} - \sum_k w'_k m'_{kj} - w'_j + w'_j(E_{TT}e'_j + 1)e'_i + w'_j W_T E_T e'_i(e'_j - 1). \tag{13}$$

5.2.2 Group price elasticity not known

If no estimate of E_{TT} is available, then equation (12) is substituted into equation (11) to give:

$$e_{ij} = m'_{ij} - \sum_k w' m'_{kj} - w'_j + w'_j[(\phi E_T - \phi W_T E_T E_T - W_T E_T)e'_j + 1]e'_i$$
$$+ w_j E_T e'_i(e'_j - 1). \tag{14}$$

This can be simplified, noting that $e_j = E_T e'_j$, to give the estimating equation:

$$e_{ij} = m'_{ij} - \sum_k w'_k m'_{kj} - w'_j + w'_j(\phi e_j + 1)(1 - W_T E_T)e'_j. \tag{15}$$

5.3 Comment on the correction for choice estimation

The difference between demand and choice is in the nature of the alternatives and the limitation on choice. Demand alternatives are those consumption vectors or market bundles that can be covered by the consumer's budget, and it is normally the ratio of prices that determines the selection of an alternative in a demand model. In contrast, the AIRUM discrete choice model implies that it is the difference between conditional cost of modes that affect choice (Train, 2003).

In the AIRUM model, the individual's utility U_j is decomposed into the sum of the observed utility V_j and an unobserved utility component e_j so that $U_j = V_j + e_j$. The probability of making choice i:

$$p_i(n) = \Pr(V_i + e_i \geqslant V_j + e_j,\ j \neq i)$$
$$= \Pr(-\lambda p_i - \beta a_i + e_i \geqslant -\lambda p_j - \beta a_j + e_j,\ j \neq i), \tag{16}$$

where p_i is the price of mode i and βa is the vector product of identified product attributes and corresponding unknown parameters. The model is conditionally homothetic, meaning that additional total income will not affect the choice probabilities (McFadden, 1981). A consequence of choice being a function of price differences rather than price ratios is that if all prices move in proportion to each other, then the choice probabilities change. In contrast, the conditional expenditure shares are left unchanged in a conditionally homothetic demand system setting when all prices move in proportion. The choice elasticities may be regarded as income-compensated homothetic conditional demand elasticities, with the correction $\sum_k w_j m'_{kj}$ being needed to move from elasticities based on price differences to elasticities based on price ratios.

6.0 An Example

To illustrate the result, using equation (13), a multinomial choice model has been estimated with a well-known revealed choice data set for travel in the Sydney–Canberra corridor collected in 1994 (Hensher, 1997; *clogit* test data set supplied with LimDep/Nlogit software). There are 210 non-business travel observations and the choices available are air, train, bus, and car. The explanatory variables are:

- p — cost of travel by mode (all modes);
- m — household income for a six-month period;
- t — travel time by mode (all modes);
- d — terminal or depot waiting time for plane, train, and bus; and
- g — number of travellers in party (air travel only: an indicator variable $d_A = 1$ if it refers to air).

The systematic utility for estimation is:

$$V_{ik} = ASC_i + \lambda_k p_{ik} + \gamma_k p_{ik}^2 + \beta_t t_{ik} + \beta_d d_{ik} + \delta_A \beta_g g_k + \delta_A \eta_A m_k + \delta_C \eta_C m_k. \qquad (17)$$

A cost-squared term is included to detect an income effect, as done by Jara-Diaz and Videla (1989). The cost terms in estimating equation (17) are not interacted with income, because the out-of-pocket costs for bus and train are higher than out-of-pocket costs for driving. The results of any such interactions would be misleading and counter to theory: households with lower income would appear to reveal a lower marginal utility of income by choosing the higher-priced alternative. Instead, two parameter estimates η_A and η_C are included to test preference for travel by air or private car for those with higher incomes. This specification follows Viton's (1985) suggested method of estimating choice elasticities with respect to income that are consistent with utility theory.

The choice model is estimated by a mixed logit. Random parameters were tested for the two price coefficients and the two income parameters. While the price coefficients had standard deviations significant at the 5 per cent level, the parameters for income did not. The parameter estimates are given in Table 1. Unconstrained standard deviations for the random parameter estimates led to a substantial proportion of respondents having positive marginal utilities with respect to price. This would mean that their direct price elasticities are positive, suggesting that restricting the standard deviation is an appropriate course of action (Hensher *et al.*, 2005). To ensure that the marginal utilities of price were of the correct sign (negative), we restricted the standard deviation of the linear price coefficient to be half the estimate for the mean and the standard deviation of the squared price to

Table 1
Mixed Logit Choice Model for Sydney–Canberra Corridor

	Estimate	*Z-value*
Cost (all)	−0.0418	(−3.72)
St. dev. cost (all)	0.021	(3.72)
Cost-squared	0.00017	(2.99)
St. dev. cost-squared	0.00005	(2.99)
Travel time (all)	−0.0044	(−3.12)
Terminal time (not car)	−0.104	(−8.47)
Group size (air)	−0.969	(−3.00)
Income × AIR	0.052	(3.16)
Income × CAR	0.040	(2.84)
AIR ASC	6.814	(5.40)
TRAIN ASC	6.101	(7.76)
BUS ASC	5.329	(7.18)
LL constants		−283.76
LL model		−172.36
Pseudo-R^2		0.408

be 0.35 of its mean estimate. Mixed logit choice elasticities do not a have closed form and are computed using a simulated method to approximate the integral (Train, 2003). The ordinary demand elasticities are computed at the individual level using equation (13) and then aggregated by the method of probability weighting.

The means of the values of travel time savings (standard deviations in brackets) are:

- air: $23.06 (10.98)
- train: $14.17 (6.91)
- bus: $11.22 (5.35)
- car: $10.71 (4.82).

These values can be compared with Bhat's (1995) HEV estimated value of $20.80 for in-vehicle time across train, air, and car in the Toronto–Montreal corridor in 1989.

The probability-weighted choice elasticities based on the estimates in Table 1 are shown in Table 2. The choice elasticity for air travel with respect to price is within the range suggested by Oum *et al.* (1992) of −0.2 to −0.6 and is comparable to the estimate of −0.4 given by Winston (1985), but less elastic than the −1.2 given in McCarthy (1997).

Table 2
Mixed Logit Probability-weighted Choice Elasticity Estimates

	Mode share elasticity with respect to price of					
Mode chosen	*Air*	*Train*	*Bus*	*Car*	*Income elasticity*	*Estimated mode share*
Air	**−0.528**	0.185	0.102	0.056	0.492	29.6%
Train	0.230	**−0.535**	0.141	0.063	−0.426	28.3%
Bus	0.360	0.371	**−0.662**	0.079	−0.605	13.5%
Car	0.172	0.147	0.060	**−0.155**	0.251	28.7%

Table 3
Ordinary Demand Elasticity Estimates Inferred from Mixed Logit Choice Elasticities

Demand for	*Ordinary elasticity with respect to price of*				*Income elasticity*	*Expenditure share*
	Air	*Train*	*Bus*	*Car*		
Air	**−0.798**	0.116	0.041	−0.005	1.226	42.1%
Train	0.149	**−0.874**	0.091	0.017	0.444	33.1%
Bus	0.217	0.259	**−0.944**	0.006	0.226	12.2%
Car	0.033	0.040	−0.013	**−0.383**	1.048	12.6%
Trip generation elasticity η_j	−0.155	−0.167	−0.093	−0.106		

The choice elasticity for rail is at the inelastic end of the range, −0.6 to −1.0, given by Oum *et al*. (1992), and the estimate of −0.66 for bus is at the elastic end of the reviewed elasticities, −0.3 to −0.7. Winston reports choice elasticities of −0.7 for rail and −1.2 for bus. Our choice elasticity of −0.155 for car is less elastic than reported in Winston (1985) and Oum *et al*. (1992), but more elastic than that given by McCarthy (1997).

The approach to inferring ordinary elasticities from the mode choice elasticities starts from the Clements (2008) theoretical result that, for an average category of goods, price elasticity of demand is approximately −0.5 and income elasticity 1.0. In our example, the category or aggregate is inter-urban travel, but, as Oum *et al*. (1992) noted, most travel demand studies are applied to individual modes and 'the effect of a price change on *aggregate traffic* is not taken into account'. Kremers *et al*. (2002) give a meta-analysis estimate of generalised transport demand elasticity on a national scale of −0.66, but this includes freight transport. With respect to income elasticity of demand, Goodwin *et al*. (2004) give a long-term value of 1.08 for total fuel consumption and 0.93 for fuel per vehicle, while Dargay (2007) gives a range of 0.85 to 1.09 for car travel. Bhadra and Kee (2008) give an income elasticity in 'thick' US airline markets of 0.6. This is slim evidence and confounded with particular modes, but the judgement is made that the deviations from the Clements (2008) base values should make price elasticity of demand for the inter-urban travel group (E_{TT}) a little more elastic at −0.6, and make income elasticity (E_T) a little less elastic at 0.9.

The share W_T of intercity travel in the household's total budget is assumed to be 1 per cent. The matrix of ordinary elasticities is calculated by the probability-weighted aggregation method using these values and the choice elasticities in Table 2. The inferred elasticities in Table 3 are the probability-weighted sums of the individual elasticities using equation (13). The estimated cross-elasticities of demand for air with respect to car cost and for car w.r.t. bus fare are effectively zero.

The ordinary demand elasticities for intercity travel (Table 3) lie within the ranges suggested by Oum *et al*. (1992): −0.4 to −2.0 for air; −0.1 to −0.5 for car; and −0.4 to −1.5 for rail and bus. Bowyer and Hooper (1993) examined the demand for holiday travel in New South Wales and made estimates of demand elasticities somewhat lower than those presented here, ranging from −0.8 to −1.5, with rail being the least elastic and demand for air the most elastic. A similar study in Spain (Coto-Millán *et al*., 1997) gave estimates of long-run own-price elasticities for intercity demand of −1.5 for air, −0.8 for rail, and −0.5 for car. The reported income elasticities were 1.3 for air and 0.9 for car. Interestingly, the authors later updated these estimates with an alternative demand model, giving an own-price elasticity of −0.5 for air and −0.3 for car (Coto-Millán *et al.*, 2007). With respect to rail travel, Wardman and Shires (2003) comment: 'One of the most consistent findings across studies . . . is an estimated fare elasticity of around −0.9 on Non London inter-urban rail flows.' Recognising that there have been many estimates of the price elasticity of demand for air travel, Brons *et al.* (2001) plotted distributions of published values. These showed modal values of −0.8 for business and −1.5 for other air travel. On the air route from Sydney to the national capital Canberra, on which our example is based, there is a high proportion of business travel and the estimated elasticity is −0.8.

The trip generation elasticities in the last row of Table 3 are derived by applying the following equation (identical to equation (A2.7)) at the individual level and forming the probability-weighted average:

$$\eta_j = e_{ij} - m'_{ij} - \omega_j m'_i. \qquad (18)$$

The derivations are based on the estimated shares, all trip and expenditure shares being substantial in this sample (last column of Table 2 and Table 3). With these shares, the example shows that the derived own-price elasticities of demand are considerably more elastic than the choice elasticities on which they are based. The cross-elasticities of demand are based on the individual results obtained in the mixed logit estimation and are meaningful estimates of the cross-relationships.

6.1 Practical importance

As an example of the importance of the transformation, the estimated ordinary demand elasticity for car trips on the Sydney–Canberra route of −0.383 (Table 3) indicates a much more substantial price response than would be assumed if the choice elasticity of −0.155 (Table 2) were treated as the demand elasticity. Similar conclusions follow from comparisons of the other own-price demand elasticities in Table 3 with the corresponding choice elasticities in Table 2: for all modes, these demand elasticities are more elastic than the corresponding choice elasticities.

7.0 Conclusion

This paper presents a method of reversing the derivation of choice from demand elasticities. The method is complicated in the general case by the need to correct for the difference between the assumptions underlying usual choice estimation methods and the assumptions of conventional demand theory.

When ordinary demand elasticities are to be inferred from choice elasticities, an estimate of the group income elasticity is required. An estimate of the aggregate demand elasticity for the group can be used as an input if it is available. However, the paper takes, as input, either a prior demand elasticity estimate or one which is approximated on the basis of Frisch's (1959) money flexibility.

The practical significance of the result is that it will prevent the errors that can result from using choice elasticity estimates as proxies for actual market demand elasticities.

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Appendix 1: Relationship between Conditional and Unconditional Elasticities under Weak Separability

This appendix is based on Smith (2011), Appendix 4.

The Slutsky term κ_{TG} relating to transport group T and a non-transport group G is defined as:

$$\kappa_{TG} = \frac{\partial H^T(P, m)}{\partial P_G} \frac{P_G}{H^T} = m^T E^*_{TG}, \tag{A1.1}$$

where H^T is the Hicksian demand function for group T, P_G is the index of group G prices, m^T is total transport group expenditure, and E^*_{TG} is the income compensated elasticity of demand for group T with respect to a change in the price index of group G. Assuming weak separability, the Slutsky term for products belonging to the two different groups is:

$$s_{ij} = \kappa_{TG} \frac{\partial x_i^T}{\partial m^T} \frac{\partial x_i^G}{\partial m^G}; \quad i \in T, \quad j \in G, \quad T \neq G. \tag{A1.2}$$

The unconditional compensated elasticity is therefore:

$$e^*_{ij} = s_{ij} \frac{p_j}{q_i} = \frac{p_j q_j}{m^G} \cdot \frac{\kappa_{TG}}{m^T} \cdot \frac{\partial x_i^T}{\partial m^T} \frac{m^T}{q_i} \cdot \frac{\partial x_i^G}{\partial m^G} \frac{m^G}{q_j} = w'_{j(G)} \frac{\kappa_{TG}}{m^T} e_i^T e_j^G. \tag{A1.3}$$

Substituting equation (A1.1), $\kappa_{TG} = m^T E^*_{TG}$, into (A1.3) gives:

$$e^*_{ij} = w'_{j(G)} E^*_{TG} e_i^T e_j^G. \tag{A1.4}$$

The corresponding ordinary elasticity is:

$$e_{ij} = w'_{j(G)} E^*_{TG} e_i^T e_j^G - w_j e_i. \tag{A1.5}$$

For any group G the sum of the j elasticities:

$$\sum_{j \in G} e_{ij} = E^*_{TG} e_i^T \sum_{j \in G} w'_{j(G)} e_j^G - e_i \sum_{j \in G} w_j = E^*_{TG} e_i^T - W_G e_i, \tag{A1.6}$$

where the last equality is due to $\sum_{j \in R} w'_j e'_{j(R)} = 1$ (that is, Engel's aggregation). If the group G is considered to be a Hicksian aggregate (that is, the prices of each good within the group remain in proportion), then the row sum of the elasticities (equation (A1.6)) is the elasticity of demand for good $i \in T$ with respect to the price index P_G. Noting that $e_i = E_T e_i^T$, the cross-price elasticity for goods within group T with respect to broad aggregate price indices are proportional to the corresponding income elasticities; that is:

$$e_{iG} = (E^*_{TG} - W_G E_T)e_i^T = E_{TG}e_i^T. \tag{A1.7}$$

Consider now the use of separable utility functions to examine transport elasticities only. All other goods are assumed to form a broad Hicksian aggregate, z. At the first stage of the budget process the cross-price elasticity for transport with respect to z is given by $E_{TZ} = e_{iz}/e'_i$. By way of homogeneity (or in terms of Slutsky substitutions), $E_{TZ} = -E_{TT} - E_T$. The cross-price elasticity for any transport alternative with respect to the price of the outside good is:

$$e_{iz} = (-E_{TT} - E_T)e'_i. \tag{A1.8}$$

Substituting equation (A1.8) into the homogeneity restriction $\sum_{k \in T} e^*_{ik} - \sum_{k \in T} w_k e_i + e_{iz} + e_i = 0$:

$$\begin{aligned}\sum_{k \in T} e^*_{ik} &= W_T E_T e'_i + (E_{TT} + E_T)e'_i - E_T e'_i \\ &= W_T E_T e'_i + E_{TT} e'_i. \end{aligned} \tag{A1.9}$$

Conditional and unconditional elasticities are related by symmetry:

$$\sum_{k \in T} w'_k e^*_{kj} = \sum_{k \in T} w'_j e^*_{jk} = w'_j \sum_{k \in T} e^*_{jk}. \tag{A1.10}$$

From equation (8) expressed in compensated terms, the ordinary demand elasticity is given by $e'_{ij} + \omega_j e'_i = e'_{ij} + (\sum_{k \in T} w'_k e^*_{kj} - w_j E_T + w'_j)e'_i$. Using the relationships (A1.9) and (A1.10):

$$\omega_j e'_i = \left(\sum_{k \in T} w'_k e^*_{kj} - w_j E_T + w'_j\right)e'_i. \tag{A1.11}$$

By symmetry (A1.10):

$$\omega_j e'_i = w'_j e'_i \sum_{k \in T} e^*_{jk} - (w'_j W_T E_T - w'_j)e'_i. \tag{A1.12}$$

Substituting the homogeneity equations (A1.8) and (A1.9):

$$\omega_j e'_i = w'_j e'_i (W_T E_T + E_{TT})e'_j - (w'_j W_T E_T - w'_j)e'_i. \tag{A1.13}$$

Rearranging:

$$\omega_j e'_i = w'_j (E_{TT} e'_j + 1)e'_i + w'_j W_T E_T e'_i (e'_j - 1). \tag{A1.14}$$

Hence equation (9) is obtained: $e_{ij} = e'_{ij} + w'_j(E_{TT}e'_j + 1)e'_i + w'_j W_T E_T e'_i (e'_j - 1)$.

Appendix 2: Derivation of Conditional Demand Elasticities from Conditional-share Elasticities

This appendix is based on Smith (2011), Chapter 5 (5.3 and 5.4).

The conditional-share income elasticity m_i' differs from the conditional income elasticity e_i' in that the condition on m_i' is the transport group quantity being held constant rather than the budget. With mode share expressed in full as $s_i = q_i/q^T$, conditional-share income elasticity is:

$$m_i' = \frac{\partial\left(\dfrac{q_i}{q^T}\right)}{\partial m^T}\frac{m^T}{\left(\dfrac{q_i}{q^T}\right)} = \left(\frac{q^T\dfrac{\partial q_i}{\partial m^T} - q_i\dfrac{\partial q^T}{\partial m^T}}{(q^T)^2}\right)\frac{m^T}{\left(\dfrac{q_i}{q^T}\right)}. \tag{A2.1}$$

Thus the conditional-share income elasticity is decomposed into a conditional income elasticity and a generation elasticity:

$$m_i' = e_i' - \frac{\partial\sum_{k\in J} q_k}{\partial m^T}\frac{m^T}{\sum_{k\in J} q_k}. \tag{A2.2}$$

Multiplying and dividing through the demand quantity q_k for each elasticity in the generation component, and noting the sum of the demands for transport, is the quantity index q^T:

$$m_i' = e_i' - \sum_{k=1}^{J}\frac{q_k}{q^T}\left(\frac{\partial q_k}{\partial m^T}\frac{m^T}{q_k}\right) = e_i' - \sum_{k=1}^{J} s_k e_k'. \tag{A2.3}$$

The conditional income elasticities are obtained from the conditional-share income elasticities by adding one to the conditional budget elasticity and subtracting it back, noting that the sum of the budget shares multiplied by the conditional budget elasticities equals one (Engel's aggregation: $\sum_{k=1}^{J} w_k' e_k' = 1$):

$$e_i' = e_i' + 1 - \sum_{k=1}^{J} w_k' e_k'. \tag{A2.4}$$

Substituting equation (A2.3) for the conditional income elasticities, the right-hand side of equation (A2.4) is written in terms of conditional-share income elasticities:

$$e_i' = \left(m_i' + \sum_{k=1}^{J} s_k e_i'\right) + 1 - \sum_{k=1}^{J} w_k'\left(m_k' + \sum_{k=1}^{J} s_k e_k'\right). \tag{A2.5}$$

Noting that pre-multiplying the generation component by the budget shares leaves it unchanged (that is, $\sum_{k=1}^{J} w_k'(\sum_{k=1}^{J} s_k e_k') = \sum_{k=1}^{J} s_k e_k'$), equation (A2.5) simplifies to:

$$e_i' = m_i' + 1 - \sum_{k=1}^{J} w_k' m_k'. \tag{A2.6}$$

Equation (12), $e_{ij}' = m_{ij}' - \sum_{k=1}^{J} w_k'(m_{kj}') - w_j'$, is proved by showing that the expression on the right of equation (12) is the conditional demand elasticity. The ordinary demand elasticity is decomposed into its conditional share and generation components:

$$e_{ij} = m_{ij}' + \omega_j(m_i') + \eta_j. \tag{A2.7}$$

Substituting the rearranged decomposition $m'_{ij} = e_{ij} - \eta_j - \omega_j m'_i$ into equation (12):

$$e'_{ij} = e_{ij} - \eta_j - \omega_j m'_i - \sum_{k=1}^{J} w'_k(e_{kj} - \eta_j - \omega_j m'_k) - w'_j$$

$$= e_{ij} - \eta_j - \omega_j m'_i - \sum_{k=1}^{J} w'_k e_{kj} + \eta_j \sum_{k=1}^{J} w'_k + \omega_j \sum_{k=1}^{J} w'_k m'_k - w'_j. \quad \text{(A2.8)}$$

Given that the sum of the expenditure shares $\sum_{k=1}^{J} w'_k$ equals one, the quantity generation elasticity η_j cancels out the equation. Simplifying the remaining terms:

$$e'_{ij} = e_{ij} - \omega_j\left(m'_i - \sum_{k=1}^{J} w'_k m'_k\right) - \left(\sum_{k=1}^{J} w'_k e_{kj} + w'_j\right). \quad \text{(A2.9)}$$

The last term in brackets is the right-hand side of equation (6) for the expenditure generation elasticity $\omega_j = (\sum_{k=1}^{J} w'_k e_{kj} + w'_j)$. Substituting ω_j in equation (A2.9) gives:

$$e'_{ij} = e_{ij} - \omega_j\left(m'_i - \sum_{k=1}^{J} w'_k m'_k\right) - \omega_j$$

$$= e_{ij} - \omega_j\left(m'_i - \sum_{k=1}^{J} w'_k m'_k + 1\right). \quad \text{(A2.10)}$$

Substituting equation (A2.6), $e'_i = 1 + m'_i - \sum_k w'_k m'_k$, in equation (A2.10) gives $e'_{ij} = e_{ij} - \omega_j e'_i$, which is equation (7), the conditional demand elasticity in terms of ordinary demand elasticity and the expenditure generation component, so that equation (12) is proved.